REPORT No. 183

THE ANALYSIS OF FREE FLIGHT PROPELLER TESTS AND ITS APPLICATION TO DESIGN

By MAX M. MUNK
National Advisory Committee for Aeronautics

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SUMMARY.

This paper, prepared for publication by the National Advisory Committee for Aeronautics, contains the description of a new and useful method suitable for the design of propellers and for the interpretation of tests with propellers.

The fictitious slipstream velocity computed from the absorbed horsepower is plotted against the relative slip velocity. It is discussed in detail how this velocity is obtained, interpreted, and used. The methods are then illustrated by applying them to model tests and to free flight tests with actual propellers.

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INTRODUCTION.

The results of tests with full size propellers in actual action have to be used in a different manner than the results of tests with model propellers. Only then the full benefit will be derived from such information. The conditions are fundamentally different in both cases and another treatment is therefore necessary.

In a well arranged model propeller test, the propeller can be considered as practically isolated, without any interference between it and adjacent objects. In that case the propeller thrust is a quantity very well defined, a quantity moreover, expected to stand in a comparatively simple and uniform relation to the characteristics of the relative motion between the propeller and the air. The torque, or the absorbed horsepower, stands in a relation necessarily less simple and uniform, since it is not only affected by the lift of the blade elements (as the chief portion of the thrust is) but in addition by their drag; and it is known that the drag of wing sections follows more erratic relations than the lift does. Hence the investigator, who aims at obtaining as clear as possible an insight into the propeller action, quite naturally turns first to the thrust as to that quantity observed in a model test which readily lends itself to an analysis. There is an additional reason of much weight, why with model propeller tests the measured thrust rather than the measured horsepower should be considered as the more important information obtained. The propeller model is always in a small scale and its tip velocity is much smaller than in flight. As a consequence there is a rather large scale effect; the results of the model test can not directly be converted into the exact figures for the full size propeller. It is now known from wing-section research that within the ordinary range the lift is much less affected by the scale than the drag is. It follows that the thrust (being chiefly produced by the lift of the blade elements) is much less affected by the scale than the torque is, which latter is noticeably influenced by the drag of the blade elements. Hence the relations obtained from model tests for the thrust are much more likely to hold true for the full-size propeller and therefore are of much more practical interest than those for the horsepower. But for one source of error, the agreement would almost be perfect; that is the elastic torsion of the blades. This error can be eliminated by using the same material for both the propeller and its model and by giving them the same tip velocity. The latter condition leads to inconvenient high R. P. M. of the model, though not to impossible ones. On the other hand, tests with propeller models running at low speed have the advantage that with them the propeller is practically rigid and maintains its shape under all conditions of test. This, it is true, may lead to a discrepancy between the performance of the propeller and that of its model, but it eliminates the effect of the elasticity entirely. Therefore, the results obtained, though less useful for the study of one particular propeller, are more useful for studying the general laws governing the aerodynamic propeller action, since this main effect has been isolated from the secondary effect of the blade distortion. It can therefore be said in general that a model propeller test is not a very good source for exact and reliable numerical data on the prototype of the model, on account of the scale effect and the elastic deformation. It is a very good method, however, for studying the propeller problem from a broad and general point of view.

ANALYSIS OF THE THRUST.

When analyzing the large series of model tests of Doctor Durand (ref. 4) I used a new analytic method, which proved useful. Using the thrust coefficient

$$C_T = \frac{T}{D^2\pi/4 . V^2\rho/2} \qquad (1)$$

where

T = Thrust
D = Diameter, $D^2\pi/4$ = Disc area
V = Velocity of flight
ρ = Density of air, $V^2\rho/2$ = Dynamic pressure of flight.

I introduced the nominal slipstream velocity v by means of the equation

$$v/V = \sqrt{C_T+1} - 1 \qquad (2)$$

and plotted the relative slipstream velocity v/V against the relative tip velocity U/V, where $U = nD\pi$ = tangential velocity component of the blade tip. The curve thus obtained was called "slip curve" and its slope

$$m = \frac{d\, v/V}{d\, U/V}$$

was called slip modulus.

The tests showed in agreement with theoretical conclusions that within the useful range the slip curve is practically a straight line. A rough and summary theoretical development gave for m the approximate expression

$$m = \frac{2.8\ S/D^2}{1 + 1.4(U/V)_o\ S/D^2} \qquad (3)$$

where

S = the entire blade area and

$(U/V)_o$ = the value of the relative tip velocity U/V, where the slip curve intersects with the horizontal zero axis, and hence nominally the thrust becomes zero.

The values of $(U/V)_o$ and m observed agreed fairly well with the values computed, as well as can be expected from the rough mathematical methods employed. The physical explanations underlying these methods are thus proved to be fundamentally sound. Since even a more elaborate computation would require a correction, it seems more expedient to restrict all computation to the simplest one imaginable and to include the influence of the blade shape and of the other propeller dimensions into the correction factor needed anyhow. These correction factors can not be obtained from model tests, but only from tests with full-size propellers in action.

ANALYSIS OF THE HORSEPOWER OR THE TORQUE.

These free flight tests consist in observing the same fundamental quantities as with the model tests, viz, the thrust, the horsepower or the torque, the number of revolutions, and the velocity of flight. The relative importance of these quantities obtained, however, is now very different. The horsepower or the torque greatly outweighs the thrust in importance, and for more than one reason. By reason of the interference between the propeller and the fuselage, the radiator, and other portions of the airplane, the thrust is now very vaguely and unsatisfactorily defined, and for this reason alone can not easily be determined nor successfully used. The tensile force transferred to the propeller through the shaft is by no means the natural correlative for the thrust, and if artificially defined to be such, the general and fundamental relations for work, efficiency, etc., do not longer hold true. It certainly will be instructive and of practical use to determine the thrust, reasonably defined, as well as can be done, but it is not expedient to assign it to the first place in the propeller investigation. The torque is much more important. A propeller is not designed for a particular thrust but for a particular horsepower to be absorbed at a certain R. P. M. The thrust is merely desired to be as large as possible. The horsepower is very exactly defined, too, and devices for measuring the torque directly can be easily imagined. The interference, indeed, has some influence on the torque, but not so much as on the thrust. The differences of the modifications of the torque when mounting one propeller on different airplanes or equipping one airplane with different propellers will even be smaller. The torque of the propeller modified by the interference is the quantity practically important, and it is therefore quite proper to include the interference effect into the correction factor used.

Since for the practice we need exact information about the horsepower, but the model tests give chiefly information about the thrust, we can only derive benefit from model tests with propellers, if we succeed to convert the general relations found for the thrust into such ones referring to the torque or to the horsepower. This can be done in a very simple way.

In a perfect fluid, without losses due to viscosity, the horsepower absorbed by an isolated propeller with constant density of thrust over the propeller disc is wholly determined if the thrust is given. For then the efficiency is

$$\eta = \frac{1}{1 + v/2V} \tag{4}$$

which is the ratio of the velocity of flight to relative velocity between air and propeller at the points of the propeller disc, hence the horsepower is

$$P = TV(1 + v/2V) \tag{5}$$

Let C_P be the power coefficient, in accordance with the thrust coefficient C_T defined by

$$C_P = \frac{P}{V^3\rho/2\ D^2\pi/4} \tag{6}$$

This power coefficient would therefore be

$$C_P = C_T(1 + v/2V) \tag{7}$$

The actual power coefficient is larger than this theoretical coefficient, as additional horsepower is required to overcome the air friction and other losses. The idea is now to treat the actual power coefficient in spite of this as in the ideal case, thus arriving at a fictitious relative slip velocity which may be denoted by w/V in order to distinguish it from the one computed from the thrust denoted by v/W. w/V is necessarily always larger than v/V, though the physical interpretation of the two quantities is the same. It will appear that the difference is not very large. Each of the two slip velocities computed from the thrust or from the horsepower can be plotted to give a slip curve. It is the torque slip curve which is important for the practice. This torque slip curve is a modification of the simpler thrust slip curve, the study of which therefore gives information on the torque slip curve. The thrust slip curve, being simpler and more readily obtained from model tests, is a good means to study the final slip

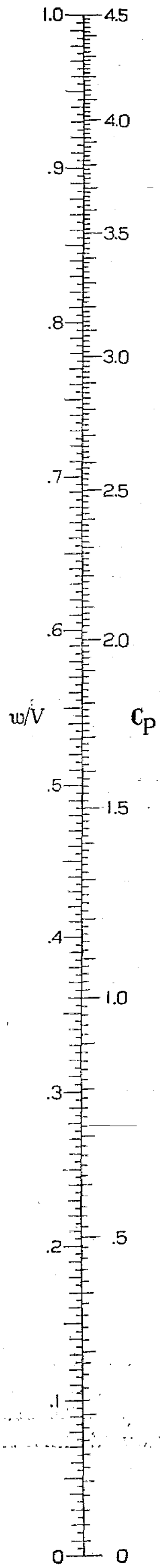


FIG. 1.

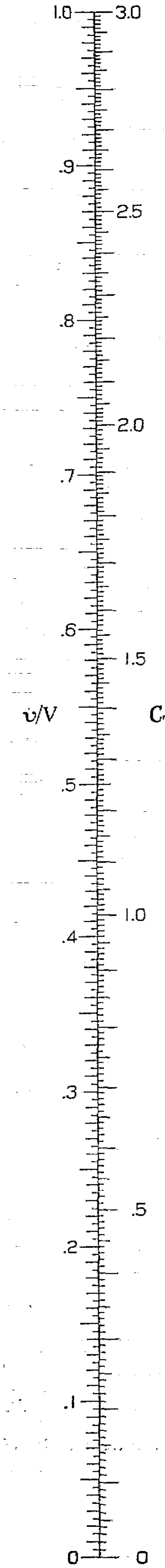


FIG. 2.

curve for the horsepower. Both curves drawn together indicate the horsepower and the efficiency, but it must be borne in mind that the propeller efficiency is a quantity as vaguely defined as the thrust is.

I proceed to establish the mathematical relation between C_P and w/V. That is now easy. Formally

$$C_T = \frac{C_P}{1 + w/2V} \quad \text{(7a)}$$

But C_T is also

$$C_T = (1 + w/V)^2 - 1 \quad \text{(8)}$$

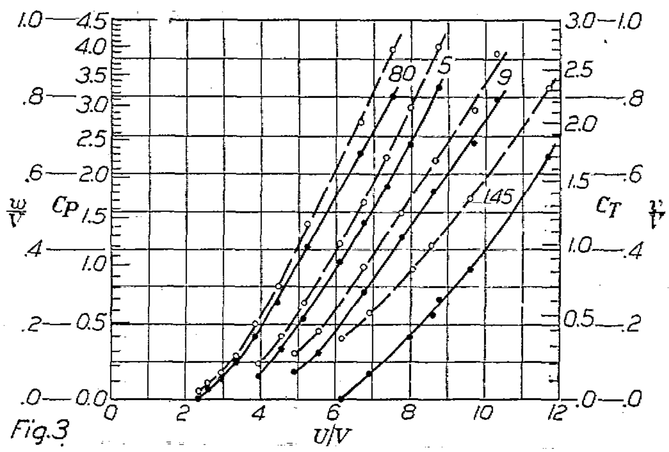
as can be obtained by inversion of equation (2). Hence

$$\left. \begin{aligned} C_P &= ((1+w/V)^2 - 1)\ (1 + w/2V) = \\ &\frac{1}{2}(w/V)^3 + 2(w/V)^2 + 2(w/V) \end{aligned} \right\} \quad \text{(9)}$$

w/V has to satisfy this equation, and is a function of C_P only. It is not convenient however to invert this equation (9). The determination of w/V from C_P can quickly be done by the use of the scale Figure 1, where w/V and C_P are plotted along the same line. Figure 2 is a similar scale for the determination of v/V from C_T. This latter scale is not quite so indispensable, as the ordinary slide rule can be used almost as quickly. It is also possible to prepare plotting paper with the vertical graduation varying as the scales in Figures 1 and 2. Then the values of the thrust coefficient and power coefficient can be plotted directly, and the slip curves are obtained without any previous conversion. In the diagrams of this paper, the magnitude of the two coefficients is indicated by scales on the sides.

APPLICATION TO A SERIES OF MODEL TESTS.

It will be helpful to illustrate the method discussed by applying it to a series of Doctor Durand's model propeller tests, though it is chiefly intended for free flight tests rather than for model tests. In Figure 3 the same slip curves as in Figure



Durand's Model Tests.

9 of reference 4 are plotted in the same way as there, and on the left side of each curve the second slip curve, computed from the horsepower instead of from the thrust, is inserted. It appears that both kinds of slip curves are of similar character and situated near to each other, but the slip curve computed from the thrust runs smoother. The space separating the

pairs of slip curves is wider at large values of U/V (small pitch) and that can be expected, for this space indicates the horsepower absorbed by the losses due to viscosity, at constant velocity of flight. This loss is larger (all other things being equal) if the number of revolutions is higher. If the pairs of slip curves would coincide, the efficiency would be

$$\frac{1}{1+w/2V}.$$

It actually is always smaller, and can be expressed by the values of a pair of v/V and w/V. For the efficiency is

$$\eta=\frac{TV}{P}=\frac{C_T V}{C_P}=\frac{(v/V)^2+2(v/V)}{\frac{1}{2}(w/V)^3+2(w/V)^2+2(w/V)}$$

$$\eta=\frac{1}{1+w/2V}\cdot\frac{v/V}{w/V}\cdot\frac{2+v/V}{2+w/V} \qquad (10)$$

At a small relative slip velocity the last factor can be neglected.

APPLICATION TO A SERIES OF FREE FLIGHT TESTS.

I proceed now to the discussion of some British free flight tests with propellers (refs. 1 and 2) which are excellently made and give full opportunity to apply this new method of analysis. The thrust of the propellers is computed from the flight characteristics observed, from experiences gained from free flight tests with the same airplane, and from such obtained from model tests, taking the increase of the drag due to the slipstream into account. As mentioned before, a perfect definition of the thrust is not possible nor necessary. The method followed by the British investigators is probably as good as any other method and gives a good indication how the slip curve computed from the thrust runs. The thrust and efficiency obtained can successfully be used only if, when used, the process of computation is inverted. Smaller changes of the airplane then will give the necessary changes of the propeller dimensions and of its performances in a satisfactory way.

The torque was determined from the R. P. M. of the engine which had been calibrated. Sometimes the objection is heard that by calibrating the engine the horsepower can not be obtained exact enough, as its magnitude depends on the condition of the engine, on the weather, and on the quality of the fuel. It certainly does, and the test would much be improved if a good torque meter could be used. On the other hand, the designer of the propeller has no more exact information on the horsepower than a calibration can give. If the correct design of a propeller would require more exact information, it would be impossible ever to design a suitable propeller. The truth is that the range of application of a propeller is broad enough to cover smaller differences of the power as caused by minor changes of the weather, of the engine, or of the fuel. It follows that measuring the power by calibrating the engine is bound to give results exact enough for practical purpose. The British tests show, moreover, that the values measured are consistent with each other, and when plotted arrange themselves along rather smooth and regular curves.

Figures 4 to 19 show the pairs of slip curves obtained from the tests. The lower curve is always v/V, the relative slip curve obtained from the thrust. These lower slip curves are remarkably straight. In the table, some characteristics of the propellers and the observed values of the slip modula for the thrust and the relative tip velocity of zero thrust are tabulated. From this latter value the mean effective angle of attack at $0.7r$ is computed by means of the equation

$$\alpha_{eff}=\cot^{-1}((U/V)_0\cdot 0.7) \qquad (11)$$

The next columns give the actual angles and the differences between the two. The actual angle is mostly smaller. The differences have to be explained by the camber effect of the blade sections. It is known from the study of wing sections that a cambered wing section produces a positive lift at the angle of attack zero, and that it has to be turned back to a negative angle of attack in order to attain to the neutral position. The camber of the average sections used with these propellers is about 10 per cent of the chord, half of this in radians, $0.05=2.9°$ is about to be expected as camber effect. The effect observed and given in the table is much smaller. There

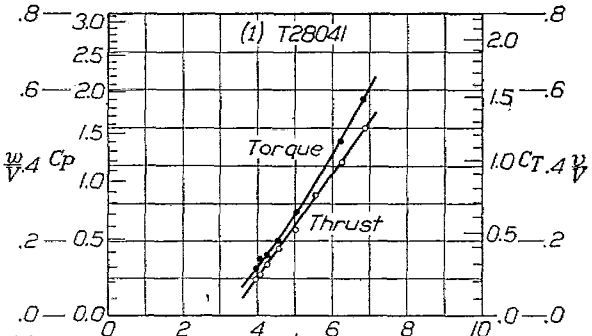


Fig.4

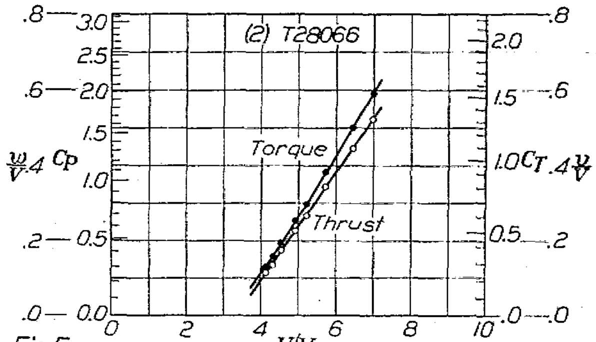


Fig.5

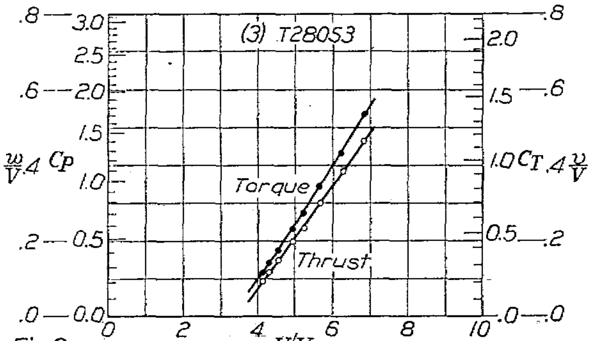


Fig.6

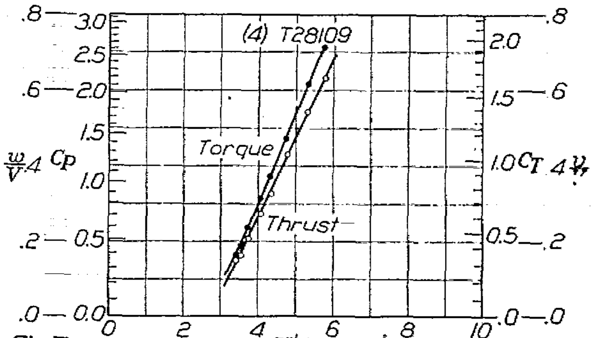


Fig.7

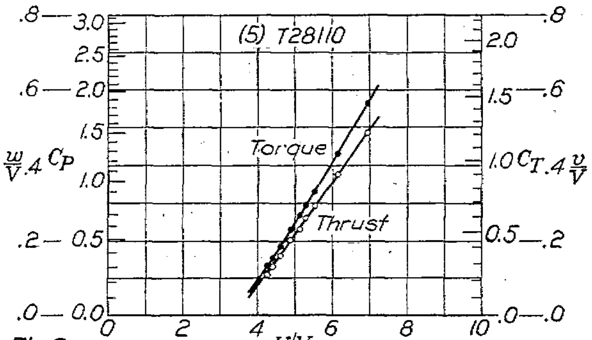


Fig.8

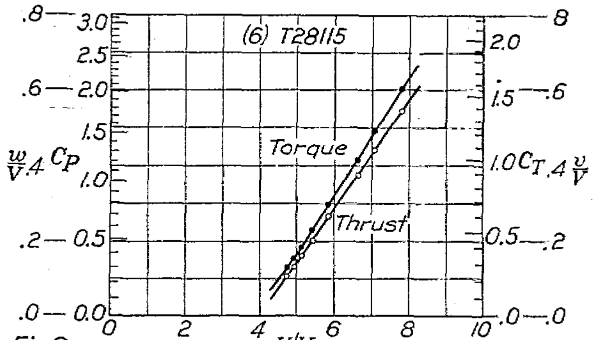


Fig.9

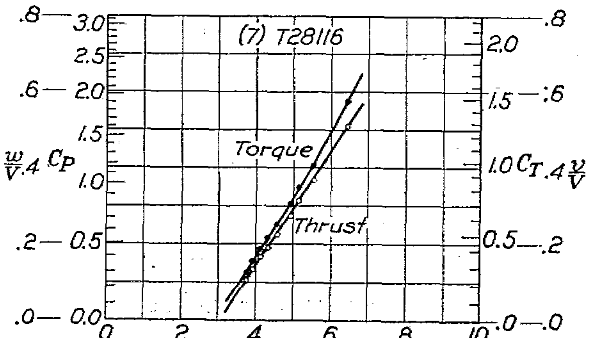


Fig.10

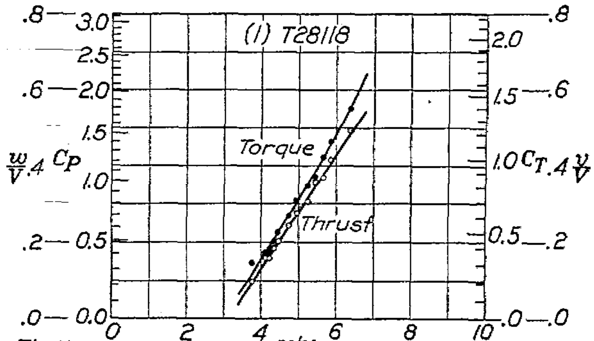


Fig.11

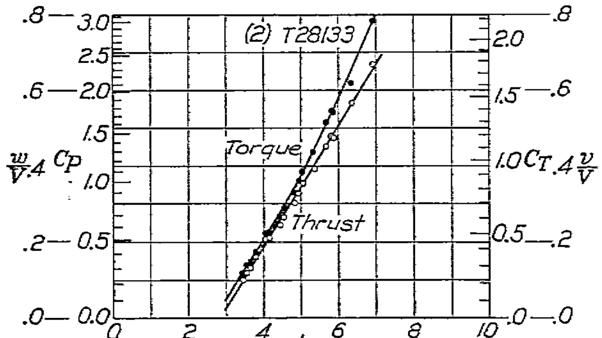


Fig.12

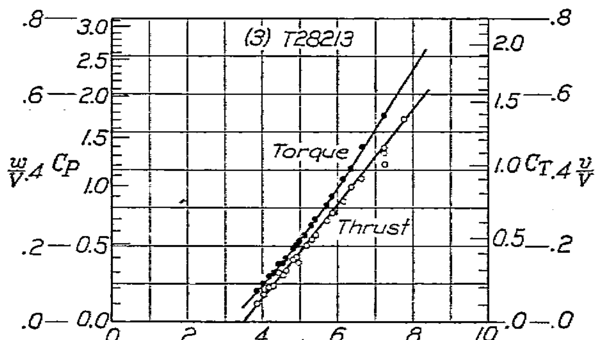


Fig.13

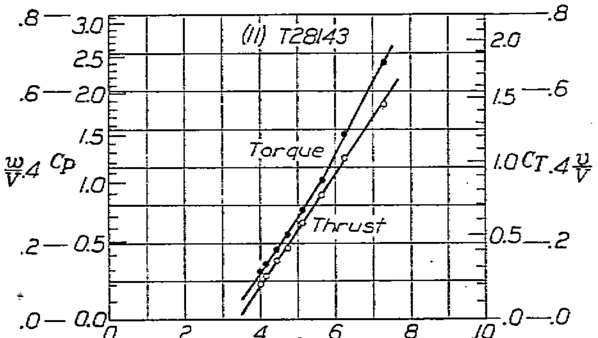


Fig.14

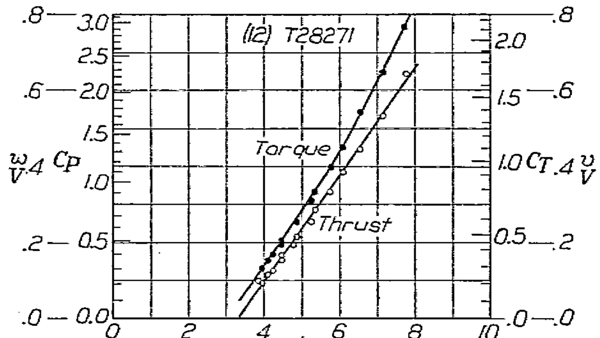


Fig.15

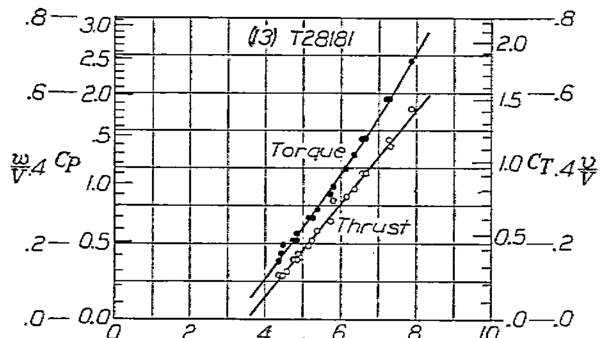


Fig.16

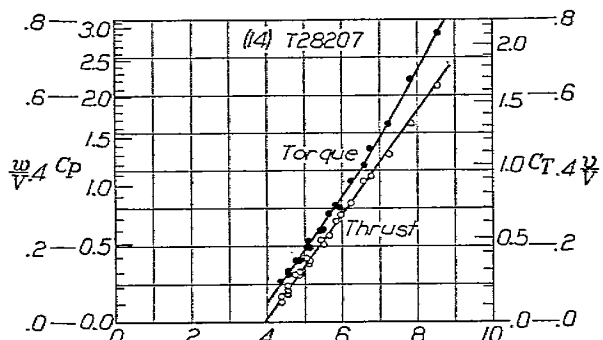


Fig.17

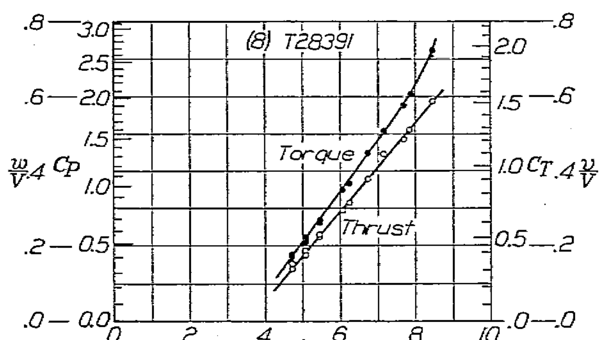


Fig.18

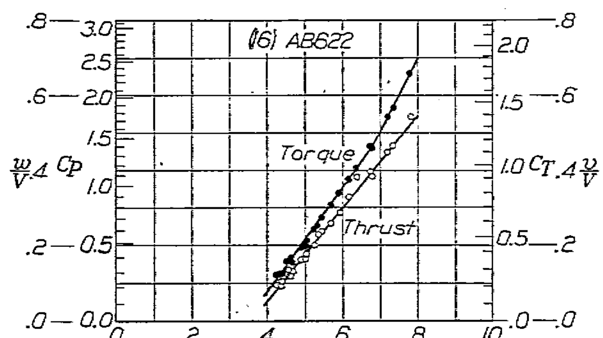


Fig.19

is then some second effect which diminishes the effective pitch. This is the elastic torsion of the blades during the flight. This assumption explains easily the difference of the propellers with respect to their values of Δ. Propellers Nos. 1, 2, 3, and 5 show practically the entire camber effect neutralized by the elastic torsion. They are similar in shape, having the maximum blade width at 0.6 of the radius. Propeller No. 2 differs from propeller No. 5 only by its blade section, the maximum camber is farther in front, giving thus rise to a smaller torque. As a consequence, the effective pitch is slightly larger. Propellers Nos. 6 and 7 have blade shapes different from the others. Propeller No. 6 has the maximum blade width nearer to the center at $0.45r$ and propeller No. 7 has about a constant blade width. Both characteristics explain a smaller torsion of the blades and therefore the larger effective pitch observed.

The slip modulus is then computed from equation 3, and in the next column of the table the slip modulus m_T obtained from the slip curve of thrust observed is tabulated. Dividing the observed slip modulus by the computed slip modulus gives a correction factor for the slip curve of thrust, which for the two blade propellers investigated lies between 1.06 and 1.12. This factor depends on the type of the propeller and probably is almost constant for each type.

The table contains in the same way the slip modulus m_P obtained from the slip curve of the horsepower and its ratio to the computed value. The necessary correction is larger in general. The slip curves for the power coincide less well with straight lines and hence the observed slip modulus is less exactly defined. I gave more weight to the lower part of the slip curve. The results vary rather much, but so do the propellers. In this respect as well as in others the two British reports are not quite consistent with each other, the results obtained with the two different motors show systematical differences. The tests are very useful as illustration of the analytical methods discussed in this paper and give some valuable information. The numerical values, however, should only be used with great care, until further research has been done along the lines indicated.

CHOICE OF THE PROPELLER DIAMETER.

The analytical method described in this paper gives quickly and conveniently a good picture of the propeller performance after experiments with it. This, however, is not all. The method used is also particularly suitable to apply the data obtained to a successful design of new propellers. I proceed to discuss this by going over the several steps the designer generally takes when laying down the dimensions of a new propeller.

The first dimension laid down is usually the diameter. Its size is determined by several independent considerations. A good efficiency under a certain condition of flight requires that then

$$\frac{U/V}{C_T} \text{ be near 30.} \quad \text{(Ref. 3.)} \qquad (12)$$

The good efficiency is obtained only, of course, if the other dimensions are chosen accordingly. Ordinarily the condition (12) gives too large a diameter. The blades become too narrow; in order to obtain a sufficient stiffness the average blade width should be at least 0.05 of the diameter and preferably larger. The second objection to too large a diameter is, that the tip velocity may become too large. U should not exceed 820 foot-seconds lest the efficiency be diminished and the stresses become too large. Often the size of the diameter is limited by the dimensions of the airplane.

The propeller finally chosen for the Hispano-Suiza engine by the British investigators, T 28066, gives a largest $\frac{U/V}{C_T}=17.2$ at an altitude of 10,000 feet and for a $U/V=4.14$. Propeller T 28207 has a maximum observed $\frac{U/V}{C_T}=31$. The former value, 17.2, is much smaller than 30, and it can be supposed therefore that one of the other two conditions rather than the efficiency was determinating and limited the size of the diameter. This is indeed the case. The engine has an unusually high R. P. M. = 2,000. The diameter chosen is 7.86 feet, giving at this R. P. M. a tip velocity of $\frac{7.86\pi 2{,}000}{60}=827$ foot-seconds. That is about the limit according to the present practice.

CHOICE OF THE BLADE AREA AND OF THE PITCH.

After having decided on the diameter, the designer can determine the blade area by formally computing the lift coefficient of the blades, supposed to be concentrated at a mean radius. Take 0.7r as the mean radius. Then the lift coefficient is approximately

$$C_L = \frac{C_T D^2 \frac{\pi}{4}}{(U/V)^2 S} \tag{13}$$

This lift coefficient should be chosen as high as possible, but not so high that the maximum lift coefficient under any conditions of flight does exceed $C_L = 0.90$ or so. The chosen propeller T 28066 gives a measured maximum lift coefficient $C_L = 0.72$, but the lift coefficient under conditions not tested may be higher and approach 0.90. At the highest speed a smallest lift coefficient $C_L = 0.37$ is recorded. That is about the lift coefficient of best efficiency.

The pitch can conveniently be determined by the use of the slip curve. This slip curve can now be computed from the conditions of flight for which the propeller is to be designed and after having decided upon the diameter and the blade area. The intersection of the slip curve and the horizontal axis gives $(U/V)_0$, which after some experience with the type of propeller used will be sufficient to compute the pitch angle and the pitch itself.

PROPELLERS OF CONSTANT REVOLUTIONS.

All propeller dimensions are then laid down preliminarily but before accepting them finally one more condition has to be examined. In order to obtain a good performance for more than one condition of flight, it is generally desirable that the torque absorbed by the propeller at the normal R. P. M. be constant. Then the engine will always give its best performance, and this advantage outweighs even a small decrease in the efficiency of the propeller. The condition of constant torque requires that the nondimensional coefficient

$$C_N = \frac{P}{U^3 \rho/2 D^2 \pi/4} = \text{constant.}$$

This coefficient or the torque itself could be computed, plotted, and it would then become apparent whether the condition of constant torque is well complied with or not. This, however, would be a very imperfect method, as it does not show directly how to choose the slip curve and thus the propeller dimensions to attain to the desired condition.

It is possible to use the slip curve itself for the examination of the constancy of the torque. Equation (14) can be transformed into one containing the relative slip velocity. For

$$C_P = C_N \ (U/V)^3$$

and by means of this equation the relative slip velocity can be obtained for different values of (u/V) and for constant C_N by the method described before. Computing it actually and plotting the curves for constant C_N each and different U/V gives a series of slip curves w/V. (Fig. 20.) These slip curves are mathematical curves of constant torque and are not generally realized by any one propeller. However, how far the actual slip curve approaches one of these curves of constant torque indicates how perfectly the condition of constant torque is complied with. The diagram can thus be used as a check of the slip curve, and more than that. The value of C_N for the particular propeller is known beforehand, as it is determined by the engine

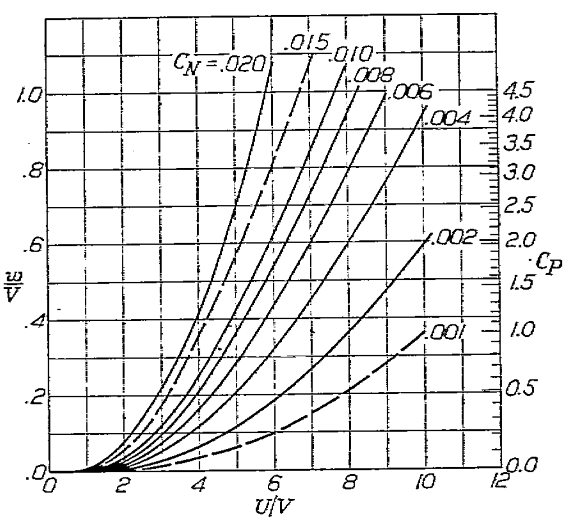


FIG. 20.—Curves of Constant C_N.

characteristics and the density of the air, and so is therefore the particular slip curve of constant torque which the actual slip curve is supposed to follow approximately. Hence, the slip curve can be drawn as a slightly curved line at the beginning and the propeller dimensions can be taken from it. That determines the lift coefficient of the blades, and if this coefficient does not come out as required for best efficiency or for the greatest thrust, or if the blades become too narrow, the diameter has to be changed or a compromise has to be made between sufficient strength, good efficiency, and small variation of the torque.

CONCLUSION.

I wished to show in this paper how the use of the slip curve is a convenient and practical way to design propellers and to study their performance after having been taken into use. In a simple and yet accurate way the method makes use of the most modern and advanced opinions of the nature of the propeller action, mechanical principles which are demonstrated by experiments to be thoroughly sound and correct. The method contains one empirical step, the conversion of the computed slip curve into the actual slip curve. The correction is not extremely large, and the computation could even be refined, and so the correction further diminished.

This important second step, the correction of the computed slip curve, should be the main subject of further experimental work with propellers. It may also be of use to study it theoretically. The free flight tests with propellers discussed in this paper do not give sufficient information on this question. They show, however, how such information can be obtained and should be obtained in the near future.

TABLE.

No.	Propeller.	Diameter.	$\frac{S}{D^2}$.	(U/V).	α eff.		α g.		Δ	
	T.	*Feet.*			°	′	°	′	°	′
1	28041	8.00	0.061	3.35	23	50	22	54		0
2	28166	7.85	.059	3.34	23	10	22	30		−20
3	28053	8.00	.054	3.40	22	45	23	00		+15
4	28109	6.50	.110	2.75	27	25	26	30		−55
5	28110	7.85	.059	3.45	22	30	22	30		0
6	28115	7.85	.063	4.00	19	40	18	24	−1	16
7	28016	7.50	.067	3.00	25	30	24	00	−1	30
8	28118	8.00	.0595	3.10	24.7	00	22.2	00	2.5	00
9	28133	7.25	.0678	2.90	26.2	00	24.1	00	2.1	00
10	28213	7.92	.0516	3.50	22.2	00	20.1	00	2.1	00
11	28143	7.50	.0675	3.30	24.2	00	19.9	00	4.3	00
12	28271	7.67	.0626	3.30	24.2	00	20.7	00	3.5	00
13	28181	7.70	.0584	3.50	22.2	00	18.5	00	3.7	00
14	28207	7.92	.0560	3.90	20.2	00	19.0	00	1.2	00
15	28391	8.00	.0467	3.60	21.7	00	20.5	00	1.2	00
16	AB622	7.84	.0527	3.60	21.7	00	18.9	00	2.8	00

No.	Propeller.	m_T computed.	m observed.	$\frac{m_T}{m\ (\text{comp.})}$.	η max.	m_P.	$\frac{m_P}{m_C}$.
	T.						
1	28041	0.133	0.141	1.06	0.74	152	1.14
2	28166	.129	.139	1.08	.73	160	1.24
3	28053	.121	.130	1.07	.73	149	1.23
4	28109	.216	.215	1.00	.76	222	1.03
5	28110	.128	.141	1.10	.74	154	1.20
6	28115	.131	.147	1.12	.73	154	1.18
7	28016	.147	.161	1.09	.76	160	1.09
8	28118	.132	.150	1.13	—	160	1.21
9	28133	.149	.156	1.04	.85	164	1.10
10	28213	.115	.123	1.07	.75	130	1.13
11	28143	.144	.143	.994	.78	147	1.02
12	28271	.136	.139	1.02	.74	145	1.07
13	28181	.127	.123	.97	.67	138	1.08
14	28207	.138	.137	1.00	.72	142	1.03
15	28391	.105	.119	1.13	—	132	1.26
16	AB622	.116	.125	1.07	.73	135	1.16